Int. J. Heat Mass Transfer. Vol. 20, pp. 994–996. Pergamon Press 1977. Printed in Great Britain

ANALYSIS OF COUNTERFLOW HEAT EXCHANGER FOR NON-NEWTONIAN FLUIDS

J. RAGHURAMAN
Department of Chemical Engineering, Monash University, Clayton, Victoria, 3168, Australia

(Received 15 October 1976)

NOMENCLATURE

B, coefficient in equation (9);
C_p, specific heat;
k, thermal conductivity;
n, flow behaviour index;
Nu, Nusselt number;
r, radial co-ordinate;
r_0, radius of the inner tube;
R, dimensionless radial co-ordinate, r/r_0;
T, temperature;
v_z, velocity in the inner tube;
$\langle v_z \rangle$, average velocity in the inner tube;
w, mass flow rate;
z, axial co-ordinate.

Greek symbols

β, eigen values of equation (10);
δ, Dirac delta function;
Λ, parameter defined in equation (1);
ϕ, eigen functions of equation (10);
ρ, density;
θ, dimensionless temperature, $(T_1 - T_{2\infty})/(T_{10} - T_{2\infty})$;
ξ, dimensionless axial co-ordinate, $\dfrac{k_1 z}{\rho_1 C_{p1} \langle v_z \rangle r_0^2}$.

Subscripts

b, bulk;
L, local;
∞, asymptotic value;
1, inner stream;
2, outer stream;
0, inlet of the inner stream.

1. INTRODUCTION

ANALYSIS of convective heat transfer in channel flow involves the solution of the appropriate energy equation with specified conditions at the channel wall. The conditions normally used are those of uniform wall temperature, uniform wall heat flux or the boundary condition of the third kind. Considerable work has appeared in the literature on these problems. In the last decade, attention was directed towards problems associated with countercurrent heat exchange with assigned temperatures of each stream at the inlet and coupling conditions in the form of temperature and flux continuity at the boundary of the two streams. Nunge and Gill [1] analysed the convective heat transfer during laminar Newtonian flow in parallel plate channels for counter flow of the two streams assuming the same physical properties in both the streams. This analysis was extended to laminar flows in double pipe heat exchangers by Nunge and Gill [2], Blanco *et al.* [3] and Stein [4], and for turbulent flow by Blanco and Gill [5]. For a class of countercurrent systems characterized by the feature that the resistance to transfer in one of the phases is negligibly small, Safonov and Potapov [6, 7] developed a method for obtaining the local and asymptotic Nusselt numbers and applied it to countercurrent heat or mass transfer during laminar flow in parallel plate channels and in circular tubes. This analysis is particularly suitable for systems in which the two streams differ sharply in their physical properties.

Hitherto all the studies have considered that both the fluid streams are Newtonian. In practice, counter flow heat exchangers where one of the stream fluids is non-Newtonian find wide applicability in polymer processing operations. It is the objective of the present work to provide a simplified analysis of the counter flow heat exchanger problem with one non-Newtonian fluid stream in which lies the major resistance to the transfer and negligible resistance in the annular stream. The non-Newtonian fluid considered is of the power law type and it will be shown that the results of Safonov and Potapov [7] for Newtonian flow and the Graetz–Nusselt problem for power law flow [8] are particular cases of the present work.

2. STATEMENT OF THE PROBLEM

Consider a concentric tube heat exchanger with a power law fluid flowing in the inner tube. The flow is assumed fully developed and laminar. Heat exchange occurs with the annular fluid flowing countercurrent to the fluid inside the inner tube. If we make the assumption that the resistance to heat transfer lies entirely on the side of the non-Newtonian fluid, then the temperature in the external stream can be taken as constant throughout the transverse section. Transfer due to axial conduction in the inner tube is considered negligible compared to convective transfer. Temperature averaged physical properties will be used in the analysis. The length of the tube is assumed to be sufficiently long that thermal equilibrium is established far away from the inlet of the inner tube. In other words, the temperature of the inner tube wall approaches the inlet temperature of the outer stream at large distances downstream of the inner fluid. This last assumption imposes a condition that

$$\Lambda = \frac{w_1 Cp_1}{w_2 Cp_2} < 1. \tag{1}$$

Thus, the investigation is limited to the range $0 \leqslant \Lambda < 1$.

3. MATHEMATICAL ANALYSIS

The energy equation for fully developed laminar flow in a circular tube with negligible axial conduction is given by

$$\rho_1 Cp_1 v_z \frac{\partial T_1}{\partial z} = \frac{k_1}{r} \frac{\partial}{\partial r}\left[r \frac{\partial T_1}{\partial r} \right]. \tag{2}$$

For a power law fluid,

$$v_z = \langle v_z \rangle \left(\frac{3n+1}{n+1} \right) \left[1 - \left(\frac{r}{r_0} \right)^{(n+1)/n} \right] \tag{3}$$

where n is the flow behaviour index and r_0 is the radius of the inner tube. From the assumption that thermal equilibrium is established far away from the inlet, the condition for constancy of heat flux through a cross-section of the exchanger can be written as

$$\pi r_0^2 \rho_1 Cp_1 \int_0^{r_0} v_z T_1 r \, dr - w_2 Cp_2 T_1|_{r=r_0} = (w_1 Cp_1 - w_2 Cp_2) T_{2\infty}. \tag{4}$$

Equations (2)–(4) are written in terms of dimensionless variables as

$$\left(\frac{3n+1}{n+1} \right) (1 - R^{(n+1)/n}) \frac{\partial \theta}{\partial \xi} = \frac{1}{R} \frac{\partial}{\partial R}\left(R \frac{\partial \theta}{\partial R} \right) \tag{5}$$

Table 1. The first eigen value β_1 for various values of n and Λ

	n				
Λ	0.2	0.4	0.6	0.8	1.0
0	2.4493675	2.5287404	2.5992317	2.6590885	2.7043642
0.2	2.2367685	2.3054181	2.3682738	2.4202747	2.4629091
0.5	1.8238731	1.8751062	1.924504	1.9658904	2.0
0.8	1.1891274	1.2194608	1.2504831	1.2768196	1.2986429
0.999	0.08970713	0.08970713	0.09223019	0.09342357	0.09392583

and

$$2\Lambda\left(\frac{3n+1}{n+1}\right)\int_0^1 (1-R^{(n+1)/n})\theta R\,dR = \theta|_{R=1}. \qquad (6)$$

The additional boundary condition is,

$$\text{at} \qquad R = 0, \quad \partial\theta/\partial R = 0 \qquad (7)$$

which results from cylindrical symmetry. The inlet conditions for the inner tube is given by

$$\text{at} \qquad \xi = 0 \quad \begin{cases} \theta = 1 (0 \leqslant R < 1) \\ = \Lambda (R = 1) \end{cases} \qquad (8)$$

for $\Lambda = 0$, equation (6) becomes, $\theta|_{R=1} = 0$, which is the condition for the Graetz problem (uniform wall temperature). Equation (5) can be solved by separation of variables. Thus,

$$\theta = \Sigma B_i \phi_i \exp\left(-\frac{3n+1}{n+1}\beta_i^2 \xi\right) \qquad (9)$$

where the eigen function ϕ_i satisfies the Sturm–Liouville equation

$$\frac{d^2\phi}{dR^2} + \frac{1}{R}\frac{d\phi}{dR} + \beta^2(1-R^{(n+1)/n})\phi = 0 \qquad (10)$$

with the boundary conditions,

$$\frac{d\phi}{dR} = 0 \text{ at } R = 0 \qquad (11)$$

and

$$2\Lambda\left(\frac{3n+1}{n+1}\right)\int_0^1 (1-R^{(n+1)/n})\phi R\,dR = \phi|_{R=1}. \qquad (12)$$

Combining equations (10) and (12), and evaluating the integral, equation (12) can be written in the alternative form as

$$2\Lambda\left(\frac{3n+1}{n+1}\right)\phi'|_{R=1} + \beta^2\phi|_{R=1} = 0. \qquad (13)$$

The eigen functions ϕ are orthogonal in the interval $0 \leqslant R \leqslant 1$ with respect to the weighting function,

$$f(R) = R(1-R^{(n+1)/n}) - \frac{(n+1)}{2\Lambda(3n+1)}\delta(R = 1) \qquad (14)$$

where δ is the Dirac delta function.

Using the initial condition and the orthogonality of the eigen functions, the constants B_i in equation (9) are obtained from the relation,

$$B_i = \frac{\phi_i|_{R=1}\dfrac{(n+1)}{2(3n+1)}(1-\Lambda)}{\Lambda\displaystyle\int_0^1 R(1-R^{(n+1)/n})\phi_i^2\,dR - \frac{(n+1)}{2(3n+1)}\phi_i^2|_{R=1}} \qquad (15)$$

for $0 < \Lambda < 1$.

For $\Lambda = 0$,

$$B_i = -\frac{1}{\beta_i^2}\frac{\phi_i'|_{R=1}}{\displaystyle\int_0^1 R(1-R^{(n+1)/n})\phi_i^2\,dR}. \qquad (16)$$

The local Nusselt number is obtained from

$$Nu_L = -\frac{2\dfrac{\partial\theta}{\partial R}\Big|_{R=1}}{\theta_b - \theta|_{R=1}} \qquad (17)$$

where θ_b is the cup mixing temperature of the inner stream and is given by

$$\theta_b = \frac{2(3n+1)}{(n+1)}\int_0^1 \theta(1-R^{(n+1)/n})R\,dR. \qquad (18)$$

Combining equations (9), (17) and (18),

$$Nu_L = \frac{2\Lambda\Sigma B_i\phi_i'|_{R=1}\exp\left(-\dfrac{3n+1}{n+1}\beta_i^2\xi\right)}{(\Lambda-1)\Sigma B_i\phi_i|_{R=1}\exp\left(-\dfrac{3n+1}{n+1}\beta_i^2\xi\right)} \qquad (19)$$

for $0 < \Lambda < 1$

and

$$Nu_L = \frac{(n+1)}{(3n+1)}\frac{\Sigma B_i\beta_i^2\phi_i'|_{R=1}\exp\left(-\dfrac{3n+1}{n+1}\beta_i^2\xi\right)}{\Sigma B_i\phi_i'|_{R=1}\exp\left(-\dfrac{3n+1}{n+1}\beta_i^2\xi\right)} \qquad (20)$$

for $\Lambda = 0$.

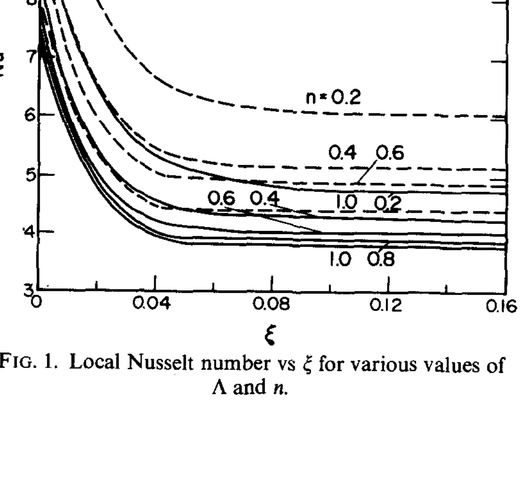

FIG. 1. Local Nusselt number vs ξ for various values of Λ and n.

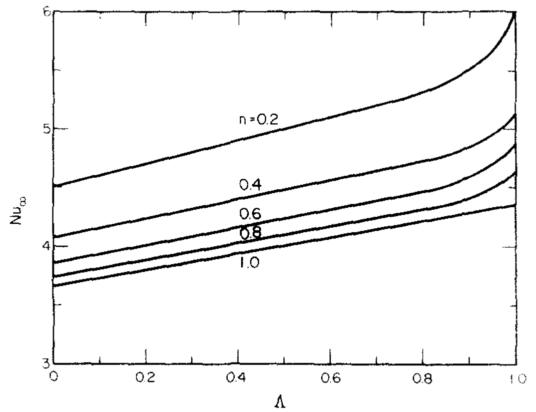

FIG. 2. Asymptotic Nusselt number vs Λ for various values of n.

At large distances downstream only the first term in the series is of major significance and hence, the asymptotic Nusselt number is given by

$$Nu_\infty = \frac{\beta_1^2(n+1)}{(1-\Lambda)(3n+1)}. \tag{21}$$

4. COMPUTATIONAL RESULTS AND DISCUSSION

Equation (10), together with the boundary conditions (11) and (13) was solved by numerical integration using the Runge–Kutta IV order method. Starting with a trial value for β, the correct eigen value was obtained by a damped Newton–Raphson iteration. The first six eigen values were computed for various values of n and Λ. For $\Lambda = 0$, the eigen values coincided with those given by Lyche and Bird [8].

For the range of values used for Λ, only the first eigen value was significantly different, the other eigen values differing only slightly. This is due to the condition given by equation (6), which was derived on the assumption that thermal equilibrium is established at large distances downstream. Table 1 gives the first eigen value for various values of the parameters.

The local Nusselt number is plotted in Fig. 1 as a function of the dimensionless axial distance ξ for two values of Λ. Since the calculations cannot be made for $\Lambda = 1$, a value of $\Lambda = 0.999$ was chosen to obtain the limiting behaviour. As ξ increased the wall temperature of the inner tube approached the outer stream temperature and there is a reduction in the driving force for heat exchange. This is reflected in the drop in the local Nusselt number from a maximum value at the entrance to an asymptotic value at larger distances down stream. It can be seen from Fig. 1 that the local Nusselt numbers are much higher as Λ increases. This is because, at larger values of Λ, the mass flow rate of the inner stream is higher resulting in a larger heat-transfer coefficient. The values of the local Nusselt numbers for $\Lambda = 0.999$ are very close to those for the case of uniform wall heat flux for $n = 1$ given by Hsu [9]. Thus, the values of Nu_L for $\Lambda = 0$ and $\Lambda = 0.999$ can be taken as the lower and upper bounds for the local Nusselt numbers. This is supported by the work of Nunge and Gill [2] who observed that $\Lambda = 1$ closely approximates the uniform wall flux behaviour both locally and asymptotically.

The effect of non-Newtonianism is also quite significant. The local Nusselt numbers increase with increasing values of the flow behaviour index n. This increase is more pronounced for n less than 0.6. For example, for $\xi = 0.05$ and $\Lambda = 0.2$, the increase in Nu_L above that for the Newtonian fluid is 3% for a fluid with $n = 0.8$, 7% for a fluid with $n = 0.6$, 16% for a fluid with $n = 0.4$ and 33% for a fluid with $n = 0.2$.

The asymptotic Nusselt number Nu_∞ is plotted against Λ for various values of n in Fig. 2. For $n = 1$, the values coincide with those of Safonov and Potapov [7]. The advantages of presenting the data in terms of the asymptotic Nusselt number are that Nu_∞ is independent of the expansion coefficients, involves only the first eigen value thereby decreasing the amount of computation and reveals directly the performance of a long tube exchanger. Since $\Lambda = 0$ corresponds to the case of uniform wall temperature and $\Lambda = 0.999$ closely approximates the case of uniform wall heat flux, the asymptotic Nusselt numbers at the two ends on the figure give the lower and upper bounds on Nu_∞. Here again, it is seen that the values of Nu_∞ are higher as n decreases. In other words, for a given long tube exchanger, given the flow rates of both the streams, the heat flux at the inner wall increases with increasing departure from Newtonian behaviour.

5. CONCLUSIONS

For a counterflow double-pipe heat exchanger with a power-law non-Newtonian fluid in laminar flow in the inner tube the local and asymptotic Nusselt numbers as a function of the flow behaviour index and the parameter Λ are presented. The cases corresponding to $\Lambda = 0$ and $\Lambda = 1$ form the lower and upper bounds for the Nusselt numbers. The Nusselt numbers increase with increasing values of Λ and decreasing values of n. The asymptotic Nusselt numbers can be used for the design of long tube exchangers.

REFERENCES

1. R. J. Nunge and W. N. Gill, Analysis of heat or mass transfer in some countercurrent flows, *Int. J. Heat Mass Transfer* **8**, 873 (1965).
2. R. J. Nunge and W. N. Gill, An analytical study of laminar counterflow double-pipe heat exchangers, *A.I.Ch.E. Jl* **12**, 279 (1966).
3. J. A. Blanco, W. N. Gill and R. J. Nunge, Computational procedures for recent analyses of counterflow heat exchangers, *A.I.Ch.E. Jl* **14**, 505 (1968).
4. R. P. Stein, Liquid metal heat transfer, in *Advances in Heat Transfer*, Vol. 3, p. 101. Academic Press, New York (1966).
5. J. A. Blanco and W. N. Gill, Analysis of multistream turbulent forced convection systems, *Chem. Engng Progr. Symp. Ser.* **63**(77), 66 (1967).
6. M. S. Safonov and Yu. T. Potapov, Diffusion entrance section with countercurrent mass transfer, *Theor. Found. Chem. Engng* **4**, 259 (1970).
7. M. S. Safonov and Yu. T. Potapov, Calculation of countercurrent heat and mass transfer in tubes, *Theor. Found. Chem. Engng* **7**, 454 (1973).
8. B. C. Lyche and R. B. Bird, The Graetz–Nusselt problem for a power-law non-Newtonian fluid, *Chem. Engng Sci.* **6**, 35 (1956).
9. C. J. Hsu, An exact mathematical solution for entrance-region laminar heat transfer with axial conduction, *Appl. Scient. Res.* **A17**, 359 (1967).